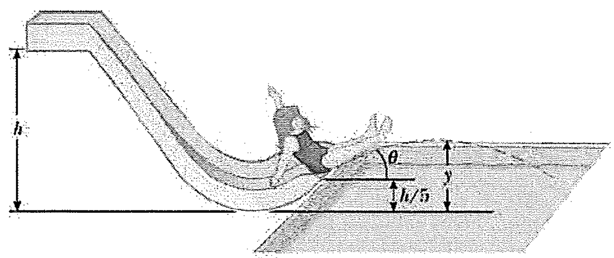
Discussion 6a : Energy and Energy Conservation

Name:______________________________

Goal: Discussing and understanding force, energy, and energy conservation.

A) A child of mass m slides from a height h along a frictionless curved slide. The bottom of the slide is a height $h_b = 0$. The slide is curved so that they actually leave the slide at a height h/5 with angle above the horizontal of θ. Gravity acts with magnitude g down.



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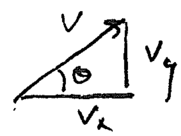
1. In terms of the given variables, h, g, θ and/or m, what is the child's speed at the bottom of the slide?

$E_{tot} = \text{const}$ so $E_{top} = E_{bot}$ or

$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$



2. What is the child's velocity (i.e., v_x and v_y) when they leave the slide?

Again $E_i = E_f \Rightarrow mgh = mg\frac{h}{5} + \frac{1}{2}mv_f^2$

$v_f^2 = \frac{8}{5}gh$ but v_f^2 is the ~~total~~ velocity so

$v_f^2 = v_x^2 + v_y^2$ and by trig $v_x = v_f\cos\theta$ $v_y = v_f\sin\theta$ so

$v_x = \sqrt{\frac{8}{5}gh}\cos\theta$ $v_y = \sqrt{\frac{8}{5}gh}\sin\theta$

3. What is the child's horizontal velocity the instant before they land in the water?

since acceleration is <u>only</u> in the y direction,

$v_{ix} = v_{fx}$ so $v_{fx} = \sqrt{\frac{8}{5}gh}\cos\theta$

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4. How high above the bottom of the slide did they reach?

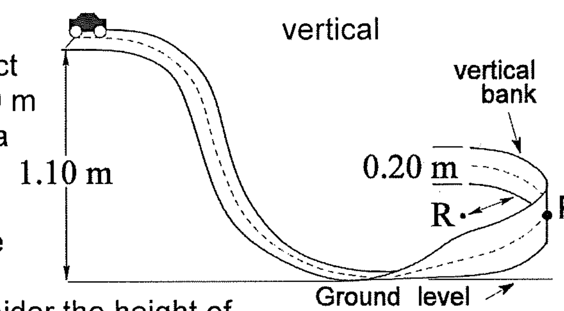
Since $\Delta v_x = 0$, we only have v_y that can change, so

$$v_{fy}^2 = v_{iy}^2 - 2g\Delta h \Rightarrow 2g\Delta h = \frac{8}{5} g h \sin^2\theta \quad \text{so}$$

$$\Delta h = \frac{4}{5} h \sin^2\theta \quad \text{but start at } h/5 \text{ so}$$

$$\text{height} = \frac{4}{5} h \sin^2\theta + \frac{h}{5} = \frac{h}{5}(4\sin^2\theta + 1)$$

B) You are designing the newest hot wheels 180° banked curved. Point P on the track forms a perfect horizontal circular arc of radius R. The track is 0.20 m wide and the car, with mass 0.10 kg, starts out at a height 1.10 m above the ground (let g = 10 m/s^2) at the center of the track and follows the path shown. The coefficient of static friction between the car and the track is 0.60. There is no rolling resistance or other sources of friction. Do not consider the height of the car.



1. What is the speed of the car at ground level?

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh} = \sqrt{2\cdot 10\cdot 1.1} = 4.69\ m/s$$

2. What is the speed of the car at point P?

$\Delta PE = \Delta KE$ so

$$mg\Delta h = \frac{1}{2}mv^2 \Rightarrow v^2 = \sqrt{2g\Delta h} = \sqrt{2\cdot 10\cdot 1} = 4.47\ m/s$$

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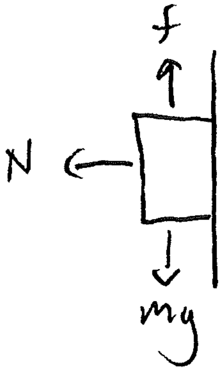
3. If the radius of curvature at point P is 0.50 m, what is the normal force of the track on the car?



x) $F = ma_c \Rightarrow N = m\frac{v^2}{r} = 0.1 \cdot \frac{20}{0.5} = \boxed{4\ N}$

4. What is the maximum radius of curvature that will keep the car from slipping down the vertical bank?

For the car to not slip, $f_s - mg = 0$

but the max $f_s = \mu_s N$ and $N = m\frac{v^2}{r}$ so

$\cancel{m}g = \mu_s \cancel{m}\frac{v^2}{r} \Rightarrow r = \frac{\mu_s v^2}{g} = \frac{0.6 \cdot 20}{10} = \boxed{1.2\ m}$

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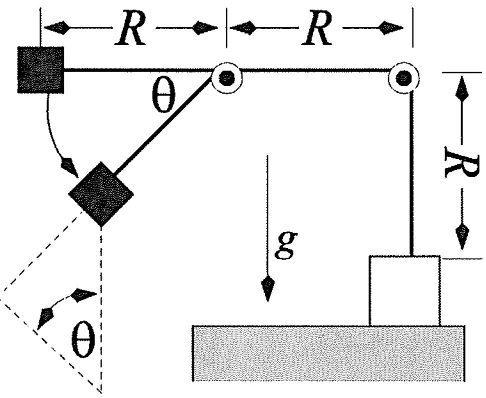
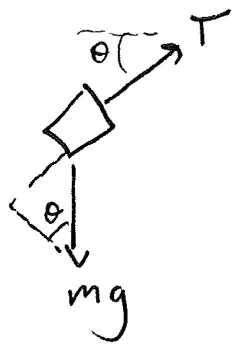
C) Two equal masses (m= 2.0 kg) are connected by a massless string (of length 3R where R = 2.0 m) which passes over two small identical massless, frictionless pulleys as shown. The first mass is resting on a table and the string rises vertically and passes over the right pulley next runs horizontally to the left over the second pulley and then on to the second mass. Gravity acts downwards at 10. m/s^2



The left mass is released from rest. At what angle θ will the right mass just begin to lift off the table?

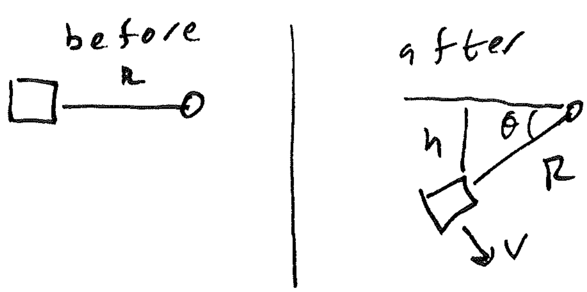


$F_{net} = T - mg\sin\theta = m\frac{v^2}{R}$

and for the block to lift, $T = mg$ so

$mg - mg\sin\theta = m\frac{v^2}{R}$

But! $\Delta PE = \Delta KE$!



so $\Delta PE = mgh = \cancel{mgR}$
$= mgR\sin\theta$

$\Delta KE = \frac{1}{2}mv^2$ so

$\frac{1}{2}mv^2 = mgR\sin\theta \Rightarrow v^2 = 2gR\sin\theta$ so

$mg - mg\sin\theta = m\frac{2gR\sin\theta}{R} \Rightarrow 1 - \sin\theta = 2\sin\theta$

$\sin\theta = \frac{1}{3}$ or $\boxed{\theta = 19.5^\circ}$