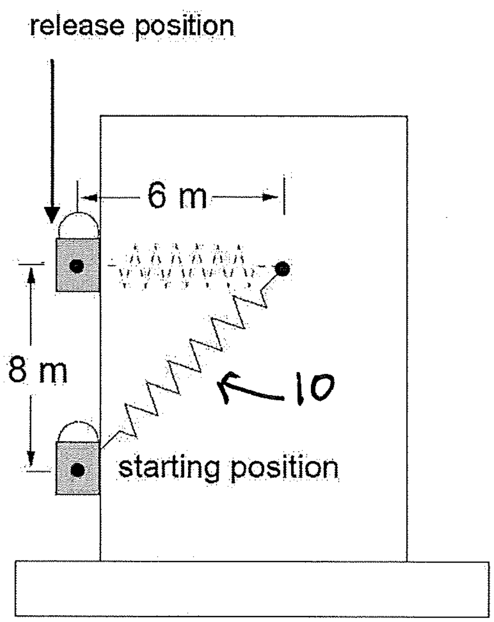
Discussion 7a : Energy and Energy Conservation

Name: Millard Fillmore

A) An ingenious friend, with way too much free time, has designed a spring-based launcher to send 2.0-kg balls of clay up vertically into the sky. The ends of the spring are designed to pivot around their respective attachment points so that the spring always behaves in a linear Hooke's Law fashion. The equilibrium length of the unstretched spring is just 2.0 m. Its spring constant is 100. N/m. The launcher starts with the spring 8.0 m below the release point and then the springs movement is stopped, as shown, with the spring length, now horizontal, a distance of 6.0 m. Each clay ball is placed in a massless cup and there are no frictional forces. Gravity acts downward with an acceleration of 10. m/s^2

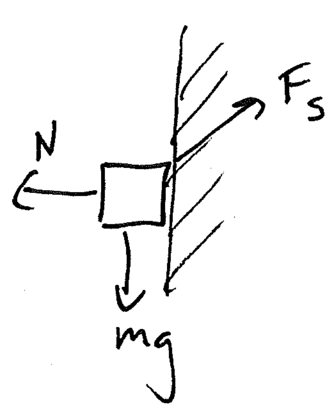


1. How much energy is stored in the spring at the **starting** position?

The spring is not in the air or moving so $PE_g = 0$ $KE = 0$ so

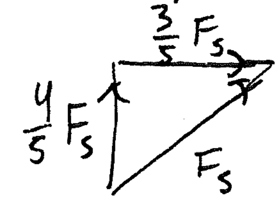
$$E_{tot} = PE_s = \frac{1}{2} k \Delta x^2 = \frac{1}{2} 100 \cdot (10-2)^2 = \frac{1}{2} 100 \cdot 64$$
$$\boxed{= 3200\,J}$$

2. What is the acceleration of the ball at the **starting** position?



Acceleration should be up so

$$F_{sy} - mg = ma$$



$$\frac{4}{5} k \Delta x - mg = ma$$

$$a = \frac{4}{5} 100 \frac{(10-2)}{2} - 10 = 320 - 10 = 310\ m/s^2$$

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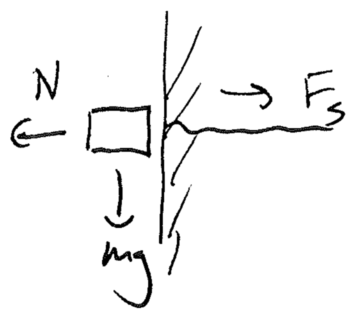
3. How much energy is stored in the spring at the **release** position?

$$PE_s = \frac{1}{2} k \Delta x^2 = \frac{1}{2} 100 (6-2)^2 = \frac{1}{2} 100 \cdot 16 = \boxed{800 J}$$

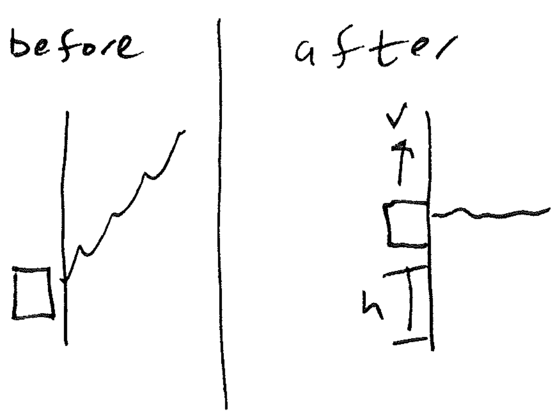
4. What is the acceleration of the ball at the **release** position?



$$\boxed{a = g} = 10\ m/s^2$$

5. What is the velocity of the ball at the release position?

We can use energy here!



We've moved up and stretched spring less and have KE now so

$$PE_{s_i} = PE_{sf} + PE_g + KE$$

$$3200 = 800 + mgh + \frac{1}{2} mv^2$$

$$2400 - mgh = \frac{1}{2} mv^2$$

$$v^2 = \frac{4800}{m} - 2gh = \frac{4800}{2} - 2 \cdot 10 \cdot 8$$

$$= 2400 - 160 = 2240 (m/s)^2 \quad \boxed{V = 47.3\ m/s}$$

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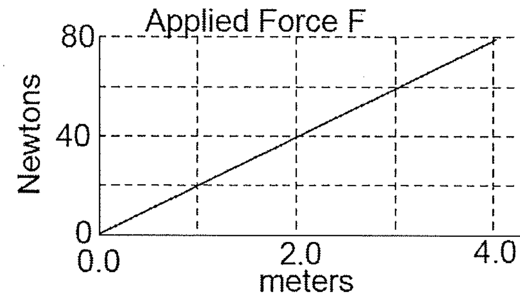
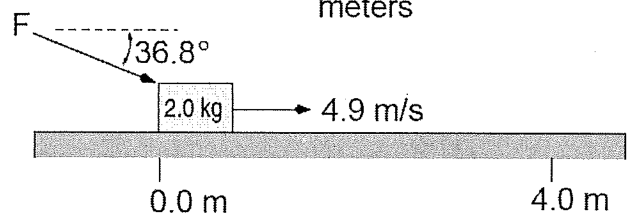
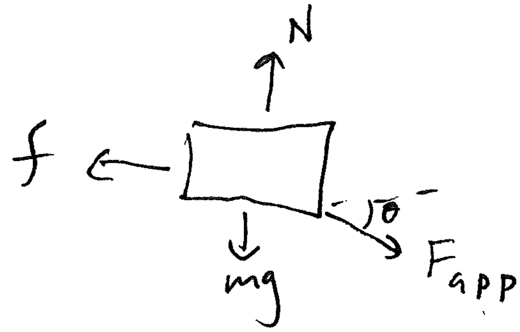
B) A 2.0-kg block, shown at right, slides across a rough surface with a constant coefficient of kinetic friction of 0.50. The block starts at x = 0 with a velocity of 4.9 m/s. Pushing the block is a force directed at 36.8 below the horizontal and whose magnitude increases with position as shown in the figure. (Assume little g is 10 m/s²).





1. What is the initial kinetic energy of the block?

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\,2\cdot(4.9)^2 \approx \boxed{24\text{ J}}$$



2. How much work is done by the applied force in the first four meters?

$$W_{applied} = \int_0^4 F_{applied}\cos\theta\, dx$$

since only $\cos\theta$ component is <u>useful</u> force

$$= \frac{4}{5}\left(\int_0^4 F_{app}\, dx\right)$$

$$= \frac{4}{5}\left(\frac{1}{2}\cdot 4\cdot 80\right) = \boxed{128\text{ J}}$$

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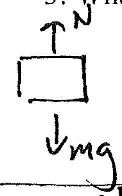
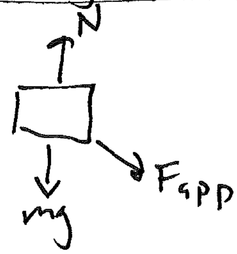
3. What is the magnitude of the friction force at 0.0 m, at 4.0 m?

0 m:



$f = \mu_k N = \mu_k mg = 0.5 \cdot 2 \cdot 10 = \boxed{10\ N}$

4 m:

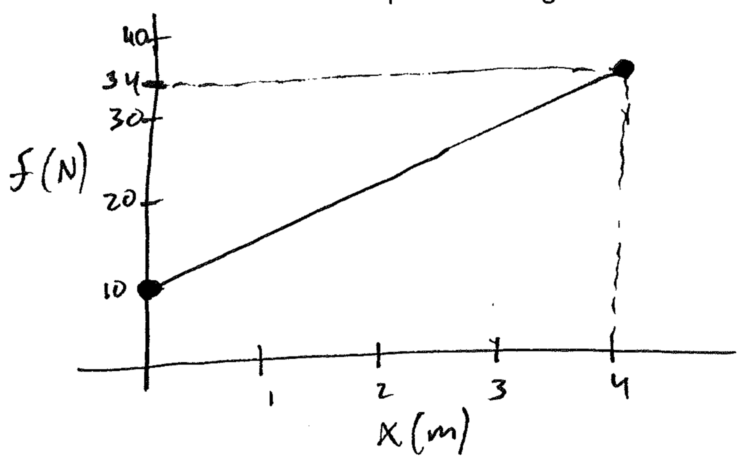


$f = \mu_k N = \mu_k (mg + F_{app} \sin\theta)$

$= 0.5 \left(2 \cdot 10 + 80 \cdot \frac{3}{5}\right)$

$= 10 + 24 = \boxed{34N}$

4. Draw a line on the force plot showing the frictional force versus distance.



$\int F_f\, dx = 4 \frac{(34+10)}{2}$

$= \boxed{88\ J}$

5. What is the speed of the block at 4.0 m?

$W_{gained} - W_{lost} = \Delta KE$

$128 - 88 = KE_f = 24$

$KE = 64J = \frac{1}{2} m v^2 = \frac{1}{2} 2 v^2$ so

$v^2 = 64\ m^2/s^2 \Rightarrow \boxed{v = 8 m/s}$