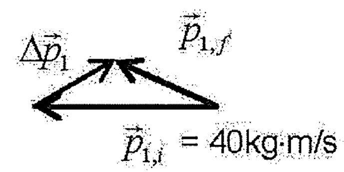
Discussion 8a : Linear momentum and Force

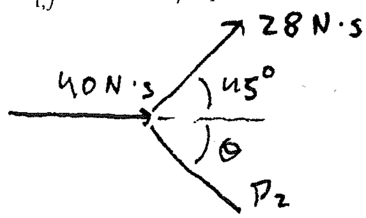
Name: Shawn Carter

A) A 2.0 kg ball (1) collides with a stationary ball (2) of mass, m = 4.0 kg. Ball 1 moves off at the angle θ_1 with velocity $\vec{v}_{1,f}$ after the collision. All angles are measured with respect to the incoming velocity vector of ball 1. In all the following cases, draw a vector that represents the momentum of ball 2 after the collision as in the example at right. Then, using $\vec{p}_{1,i}$ and the x and y components of $\vec{p}_{1,f}$ determine $\vec{p}_{2,f}$ and $\vec{v}_{2,f}$. Find both magnitude and direction (angle) in each case.



So $\Delta\vec{p}_2 = -\Delta\vec{p}_1$, $\Delta\vec{p}_2 =$ ↙

1) $\vec{v}_{1,f}$ = 14m/s, θ_1 = 45°. Make your drawing to scale.



$$\begin{pmatrix} 40 \\ 0 \end{pmatrix} = \begin{pmatrix} 28\cos 45° \\ 28\sin 45° \end{pmatrix} + \begin{pmatrix} P_x \\ P_y \end{pmatrix}$$ so

$P_x = 40 - 19.8 = 20.2$ N·s

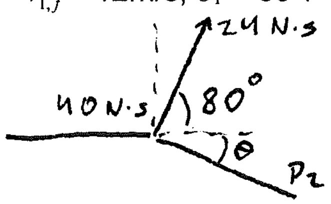
$P_y = -19.8$ N·s so

$P_{tot} = \sqrt{20.2^2 + 19.8^2} = \boxed{28.3 \text{ N·s}}$

$\theta = \arctan\left(\frac{19.8}{20.2}\right) = \boxed{44.4° \text{ below}}$

$\vec{V}_{2f} = \begin{pmatrix} 5.05 \\ -4.95 \end{pmatrix}$ m/s or $\boxed{|V| = 7.07 \text{ m/s}}$

2) $\vec{v}_{1,f}$ = 12m/s, θ_1 = 80°. Make your drawing to scale.



$$\begin{pmatrix} 40 \\ 0 \end{pmatrix} = \begin{pmatrix} 24\cos 80° \\ 24\sin 80° \end{pmatrix} + \begin{pmatrix} P_x \\ P_y \end{pmatrix}$$ so

$P_x = 40 - 4.17 = 35.8$ N·s

$P_y = -24\sin 80° = -23.6$ N·s

$P_{tot} = \sqrt{35.8^2 + 23.6^2} = \boxed{42.9 \text{ N·s}}$

$\theta = \arctan\left(\frac{23.6}{35.8}\right) = \boxed{33.4° \text{ below}}$

so $\vec{V}_{2f} = \begin{pmatrix} 8.95 \\ -5.9 \end{pmatrix}$ m/s or $\boxed{|V| = 10.7 \text{ m/s}}$

3) Which of the above case is the most elastic collision? Show your calculation and reasoning.

Most elastic means least energy lost $\Rightarrow E = \frac{p^2}{2m}$

$KE_f^{(1)} = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} = \frac{28^2}{2\cdot 2} + \frac{(28.3)^2}{2\cdot 4} = 296$ J

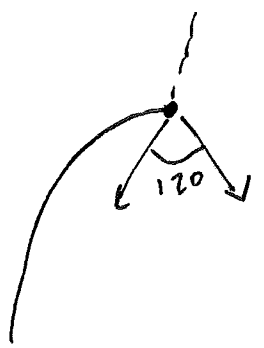
$KE_f^{(2)} = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} = \frac{24^2}{2\cdot 2} + \frac{(42.9)^2}{2\cdot 4} = 374$ J

so (2) lost the least energy so more elastic!

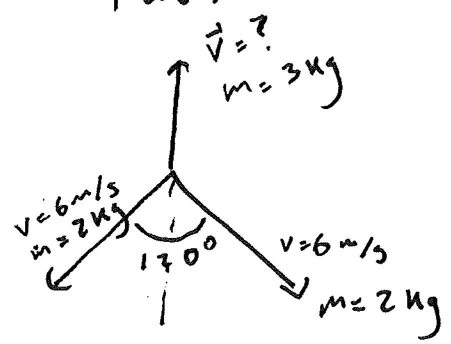
Name: ____________________________

B) You are a rocket designer testing a new escape system. You launch the 7.0 kg assembly straight up in the air. It flies up 50 m and, at the top of its trajectory, the prototype system explodes into three fragments. Two of the fragments are equal in mass (each of 2.0 kg) and leave explosion point with equal speeds (of 6.0 m/s). Both of these pieces head towards the ground and there is an angle of 120° between their respective velocity vectors. The plane formed by these two velocity vectors is perfectly vertical. In addition, each of the two pieces hits the level ground at the same moment in time (you should ignore the effects of air resistance).

To what height does the third piece rise?



We are at the top of its trajectory so $V_i = 0$ so $p_i = 0$ so $\Delta p = 0$ or $\vec{P}_{tot\,f} = 0$!

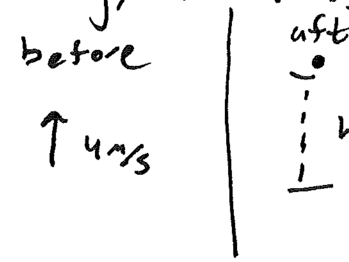


$$\begin{pmatrix} 12 \cdot \sin 60 \\ -12 \cos 60 \end{pmatrix} + \begin{pmatrix} -12 \sin 60 \\ -12 \cos 60 \end{pmatrix} + \begin{pmatrix} P_x \\ P_y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

so $P_x = 0$

$P_y = 2 \cdot 12 \cdot \cos 60 = 12\,N \cdot s$

since $m = 3kg$, $V = 4$ m/s so



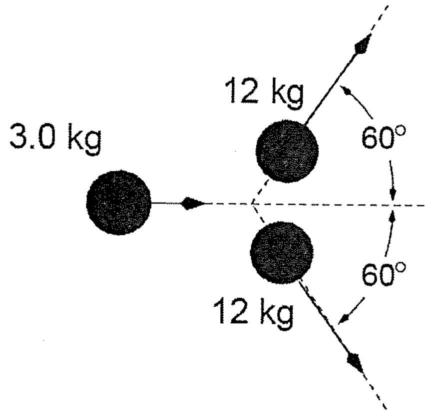
so $\frac{1}{2}mv^2 = mgh$ or

$h = \frac{v^2}{2g} = \frac{4^2}{2 \cdot 10} = 0.8\,m$

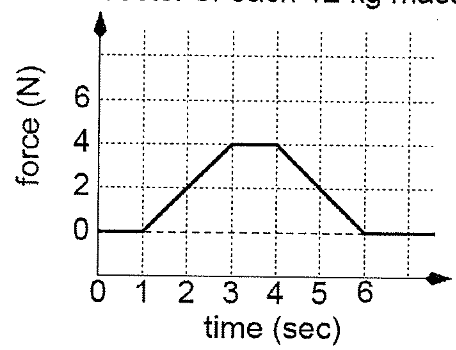
so $\boxed{h_{tot} = 50.8\,m}$

Name: ______________________________

C) Three masses, as shown below, are placed on a horizontal frictionless table. Initially the two larger masses are at rest and the 3.0 kg mass is traveling to the right. There is a collision and afterward the 3.0 kg mass is seen to be at rest while the two 12 kg masses move along the indicated paths. Force sensors, in each 12 kg mass, produce identical plots as shown (for forces along the respective velocity vectors).



Force vs. time along the velocity vector of each 12 kg mass

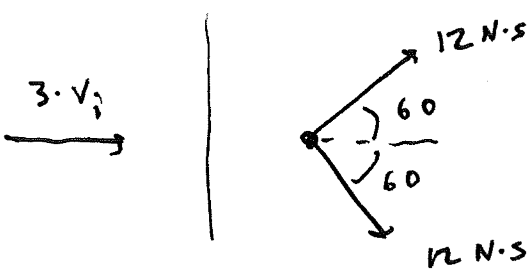


(1) What is the final speed of a 12 kg mass?

$\int F\,dt = \frac{(3+1)}{2}\,4 = 12\ \mathrm{N \cdot s} = \Delta p$ but $p_i = 0$ so

$p_f = 12\ \mathrm{Ns} = m v_f = 12\, v_f$ so $\boxed{v_f = 1\ \mathrm{m/s}}$

(2) How much mechanical energy (if any) was lost in the collision?



$$\begin{pmatrix} 3v \\ 0 \end{pmatrix} = \begin{pmatrix} 12\cos 60 \\ 12\sin 60 \end{pmatrix} + \begin{pmatrix} 12\cos 60 \\ -12\sin 60 \end{pmatrix}$$

$3v = 2 \cdot 12 \cdot \cos 6 = 12\ \mathrm{N.s}$ so

$v_i = 4\ \mathrm{m/s}$ so

$\Delta E = E_{lost} = \frac{1}{2} \cdot 3 \cdot 4^2 - 2\left[\frac{1}{2} \cdot 12 \cdot 1^2\right] = 24 - 12 = \boxed{12\ \mathrm{J}}$