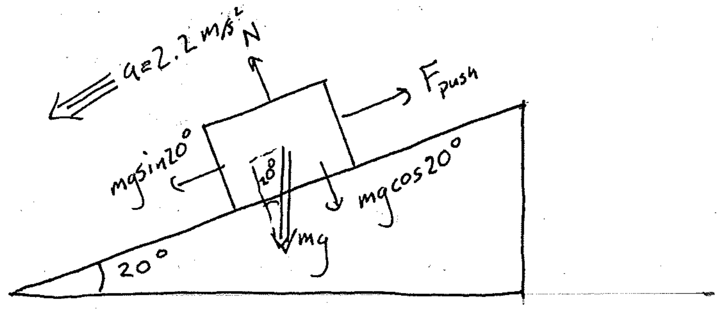
Discussion 8b : Review of chapters 5 - 8

Name: Ida B. Wells.

A) A 3.0-kg block slides on a 20° inclined plane. A force acting parallel to the incline is applied to the block. The acceleration of the block is 2.2 m/s^2 down the incline. Start with a drawing.



1) What is the applied force if the incline is frictionless?

Acceleration is down the plane, so

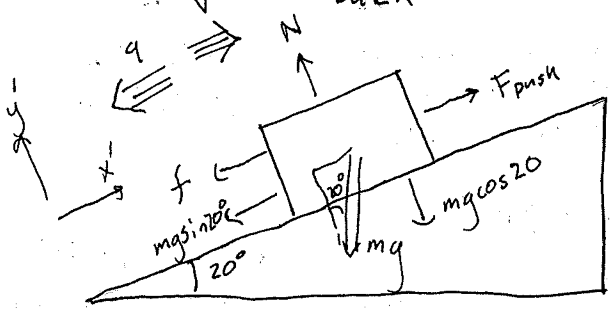
$mg\sin 20° - F = ma \Rightarrow mg\sin 20° - ma = F$

$F = 3\cdot 10\cdot \sin 20° - 3\cdot 2.2 = \boxed{3.66\ N}$

(If you had force in opposite direction, $F = -3.66N$, so sign tells us we put the arrow on wrong!)

2) What is the applied force if the incline has a kinetic friction of 0.1? $\boxed{\mu_K = 0.1}$

Edwin (ty) pointed out this is a bit ambiguous since friction depends on direction of motion. I will do up the ramp here and down the ramp on the back



$f = \mu_K N$ so we need N!

y': $N = mg\cos 20°$

x': $mg\sin 20° + f - F = ma$ or

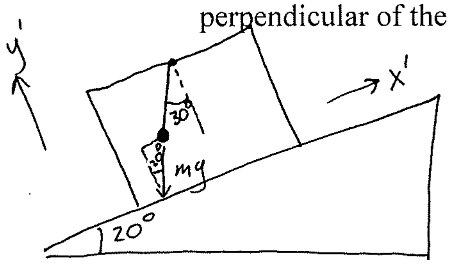
$F = mg\sin 20° + \mu_K mg\cos 20° - ma$

$= 3\cdot 10\cdot \sin 20° + (0.1)\cdot 3\cdot 10\cdot \cos 20° - 3\cdot 2.2$

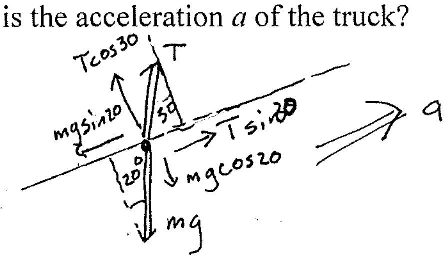
$\boxed{= 6.48\ N}$

Name: ______________________________

B) A truck with constant acceleration *a* goes up a hill that makes an angle of 20° with the horizontal. A small sphere of mass *m* is suspended from the ceiling of the truck by a light string. If this pendulum makes a constant angle of 30° with the perpendicular of the ceiling, what is the acceleration *a* of the truck?



or



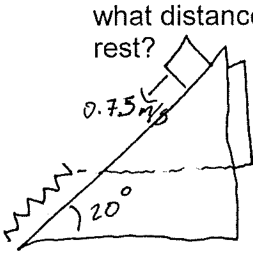
so in y' direction, $a=0$, but in x' direction, a is acceleration since the ball moves with the truck. The truck transfers that acceleration with the tension!

$$y': \; T\cos 30° - mg\cos 20° = 0 \quad \text{or} \quad T = mg\frac{\cos 20°}{\cos 30°}$$

$$x': \; T\sin 30° - mg\sin 20° = ma \implies mg\frac{\cos 20°}{\cos 30°} - mg\sin 20° = ma(\cos 20° \tan 30° - \sin 20°)$$

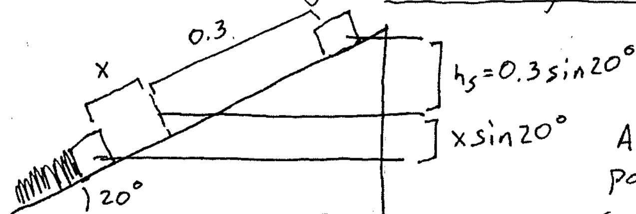
$$\text{so} \quad a = g(\cos 20° \tan 30° - \sin 20°) = 10(0.2005) = \boxed{2.01 \text{ m/s}^2}$$

C) An inclined plane with a 20° angle to the horizontal has a spring at the bottom of the incline mounted parallel to the incline plane (k=500 N/m). A block of 2.5 kg is placed on the plane at a distance of 0.3 m from the spring. From this position the block is projected downward toward the spring with a speed of 0.75 m/s. By what distance is the spring compressed when the block momentarily comes to rest?



We can see the spring has to go $h_s = 0.3\sin 20°$ to get to the spring. Then the block compresses the spring by continuing to move down the track

still lose height in compression



so $h_{tot} = (0.3 + x)\sin 20°$

All the kinetic and gravitational potential energy is converted into spring potential energy.

$$PE_g + KE = PE_s \implies mg(0.3+x)\sin 20° + \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2$$

$$(2.5)(10)(0.3+x)\sin 20° + \tfrac{1}{2}(2.5)(0.75)^2 = \tfrac{1}{2}(500)x^2 \quad \text{or}$$

$$250x^2 - 8.55x - 3.268 = 0 \quad \text{so} \quad x = 0.133\text{ m} / -0.0099\text{ m}$$

We take the positive value (x on our diagram is positive) so $\boxed{x = 0.133 \text{ m}}$

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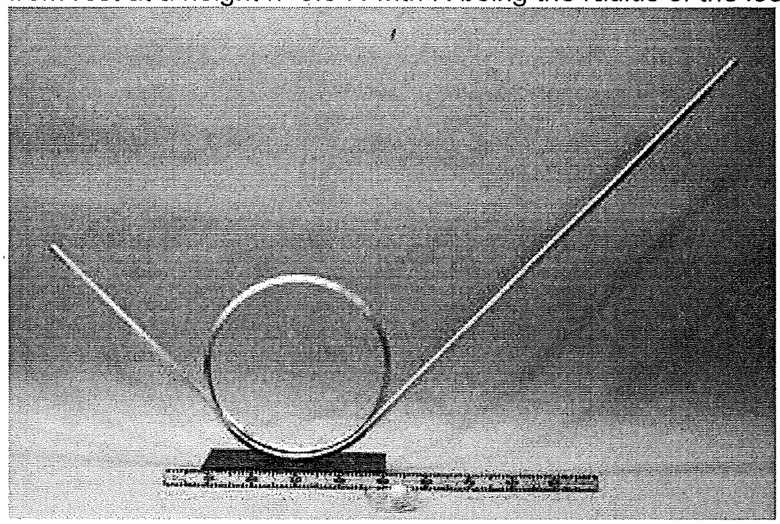
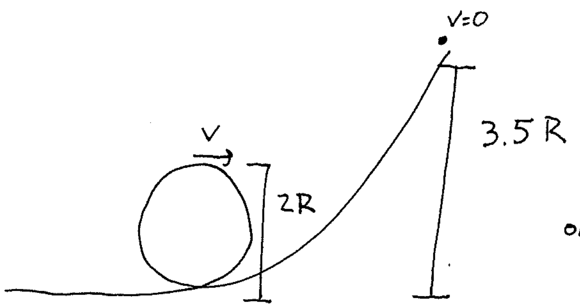
D) A ball rolls down a loop-the-loop (see picture) without friction. The ball is released from rest at a height h=3.5 R with R being the radius of the loop.



1. What is its speed at the top of the loop?



$E_i = (3.5\,R)\,mg$

$E_f = \frac{1}{2}mv^2 + mg(2R)$ so

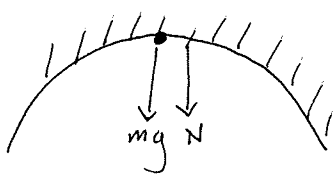
$\frac{1}{2}mv^2 = mg(3.5R) - mg(2R) = 1.5\,mgR$

or $v^2 = 2(1.5\,R)\,g$ so $\boxed{V = \sqrt{3gR}}$

$= \sqrt{30}\sqrt{R}$

$= 5.48\sqrt{R}$

2. How large is the normal force on the ball at the top of the loop if the ball's mass is 5 g?

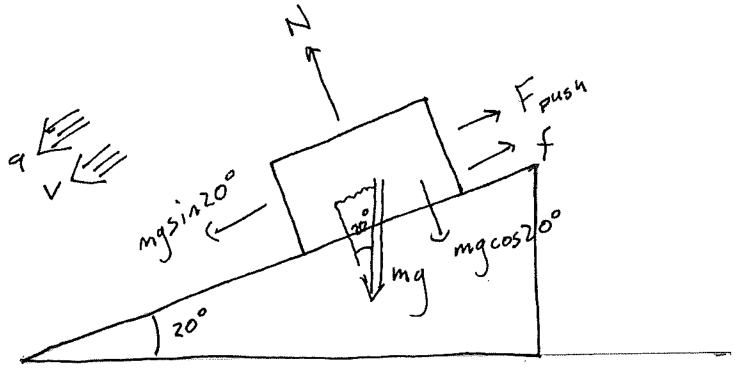


Normal stops the block from going through the track, so it is also inwards, so since circular motion, centripedal acceleration

$N + mg = m\frac{v^2}{R} = m\frac{3Rg}{R}$ so

$N = 3mg - mg = 2mg = 2(0.005)(10) = \boxed{0.1\ N}$

A) cont.



Since velocity changed direction, so does our friction, so

$N = mg\cos 20°$

$mg\sin 20° - F - \mu_k mg\cos 20° = ma$

$F = mg\sin 20° - \mu_k mg\cos 20° - ma$

$3 \cdot 10 \cdot \sin 20° - (0.1)\, 3 \cdot 10 \cdot \cos 20° - 3 \cdot 2.2$

$\boxed{= 0.842\,N}$