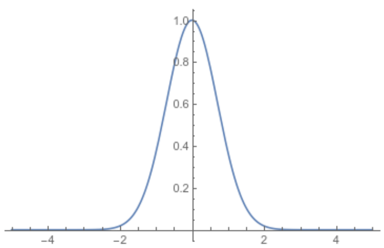
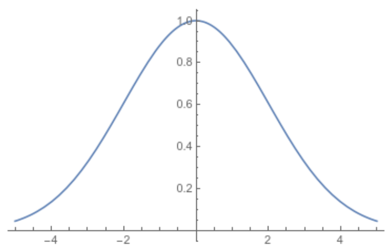
1 Overview

The biggest thing to remember when talking statistics in a lab setting is that everything you measure is in a bell-curve (normal or gaussian are other names) distribution. When we take data, it's as if we're throwing a dart and the bell-curve just tells us where the dart is most likely to hit. The usual spot is near the middle of the curve. With the bell-curve dart board, we're assuming we're all decent shots, so throwing a dart far away from the bulls eye or middle is pretty unlikely.

The worse of a shot we are, the more likely we're going to throw farther away from the center, so our gaussian is going to become wider to reflect that. With this analogy, how good of a shot is the error or width of the bell-curve. The worse a shot, the wider the curve, the better a shot, the narrower the curve.



(a) Narrow Bell-Curve (Good shot)



(b) Wide Bell-Curve (Bad shot)

Now lets say we saw someones dart board, and they got a bunch of their darts near -3. We have to ask ourselves, did the dart-thrower intentionally throw to the left, or is the dart thrower just a bad shot and got unlucky with all their shots?

This is the basic question of statistics. If we have very small error, then the probability of getting a -3 or any number less than -3 is almost 0, since we can see the narrow graph is basically 0 at that point. For the Wide graph, the probabiliy of getting a -3 or less is small, but not unreasonable, or it's well within likelihood that the person is a bad shot and was really unlucky this time. That's the basics of the error analysis. The math for doing it mainly comes from looking at a table, so here is a worked example to help you out

2 Worked Example

Let's say we've done the Gravity lab, and after calculating, we find our value is:

$$g_{measured} = 9.84 \pm 0.01 \frac{m}{s^2}$$

we plug this into our equation for how many sigma we are away from the accepted $9.81 \frac{m}{s^2}$:

$$\text{(Sigma Away)} = \frac{\text{(Accepted Value)} - \text{(Your Value)}}{\text{(Error)}} \tag{1}$$

$$z = \frac{x - \mu}{\sigma} \tag{2}$$

$$z_g = \frac{9.84 - 9.81}{0.01} = 3 \tag{3}$$

So we are 3 sigma away. Next, we look at the probability of that happening from our table:

# σ away	Probability to get > #σ away
0	100%
0.5	61.8%
1	32.8%
1.5	23.4%
2	4.6%
2.5	1.3%
3	0.3%

So, looking at the table, there is a 0.3% chance this happened by luck, or we can be confident up to 99.7% that our result is not a "real" measurement of the acceleration of gravity, but there is something causing it to be slightly higher, ie a systematic uncertainty. So for

> We found our value of $g = 9.84 \pm 0.01 \frac{m}{s^2}$, which is 3σ away from the accepted value of $9.81 \frac{m}{s^2}$, so we can say there is a systematic uncertainty with 99.7% confidence. The likely source of uncertainty came from a miss-timing of the sparker, or possibly we underestimated our error.

See how the measured and accepted value are compared, and since there was a suggested systematic uncertainty, we tried to explain it under the limitations of the lab. If you get a value you that suggests a systematic uncertainty, and you came up with some ideas for what they were, in the conclusion, say how you would test these ideas, e.g.

> Our value was shown to be outside the accepted value with 99.7% confidence. If the error was caused by miss timing of the sparker, we might connect our paper to a cart that also has it's position measured by the PASCO motion sensor. Because we know the PASCO sensor to be fairly accurate, we would have a measurement to compare to.

Now we've covered all our bases: compared value to accepted value, seen if it's off, explained why it may be off, and said how to test it. If it was in the accepted range, we could stop at saying we are within the accpeted value.

3 Cheat Sheet

- Find how many sigma away you are:

$$z = \frac{x - \mu}{\sigma} \tag{4}$$

- Check table to see if you are significantly far from the accepted (usual cut-offs are 95% or 99%.
- Tell if there is a systematic uncertainty (in or out of range determines this!)
- Explain what could caused the systematic (IF THERE IS ONE)
- Explain how to test this systematic you proposed (IF THERE WAS ONE)